مدیریو

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Nonperiodic Preventive Maintenance for Repairable Systems

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Abstract: As a complex system with multiple components usually deteriorates with age, preventive maintenance (PM) is often performed to keep the system functioning in a good state to prolong its effective age. In this study, a nonhomogeneous Poisson process with a power law failure intensity is used to describe the deterioration of a repairable system, and the optimal nonperiodic PM schedule can be determined to minimize the expected total cost per unit time. However, since the determination of such optimal PM policies may involve numerous uncertainties, which typically make the analyses difficult to perform because of the scarcity of data, a Bayesian decision model, which utilizes all available information effectively, is also proposed for determining the optimal PM strategies. A numerical example with a real failure data set is used to illustrate the effectiveness of the proposed approach. The results show that the optimal schedules derived by Bayesian approach are relatively more conservative than that for non-Bayesian approach because of the uncertainty of the intensity function, and if the intensity function are updated using the collected data set, which indicates more severe deterioration than the prior belief, replacing the entire system instead of frequent PM activities before serious deterioration is suggested. © 2010 Wiley Periodicals, Inc. Naval Research Logistics 57: 615–625, 2010

Keywords: Bayesian; nonhomogeneous Poisson process; preventive maintenance; repairable system

1. INTRODUCTION

A repairable system is the one that can be restored to its operating condition by some repair processes when a breakdown occurs instead of being replaced. Many complex systems, such as vehicles, computers, and air conditioners, are repairable; they deteriorate with age. Preventive maintenance (PM) is normally implemented to improve reliability during the lifetime of a repairable system. However, as frequent PM activities often result in high maintenance costs and poor system availability, the determination of a proper PM policy is a trade-off between system reliability and maintenance costs. The decision of an optimal PM policy involves numerous uncertainties, such as the initial status of the system, the expected system lifetime, the repair cost, and the breakdown costs. These uncertainties would increase the difficulty in determining a proper PM policy.

Many studies have modeled and analyzed the deterioration of repairable systems based on the assumption that the breakdown process follows a nonhomogeneous Poisson process (NHPP) (see, e.g., [4, 6, 28]). The Weibull power law intensity function is often used for NHPP because of its flexibility [1]. PM is usually performed to slow down the deterioration process of repairable systems. In most cases, PM policies can be divided into two main categories: periodic and nonperiodic. Periodic PM can be defined as a system which is maintained at integer multiples of some fixed period; nonperiodic PM performs the same tasks but without fixed periods. Generally, periodic PM is more convenient to schedule than nonperiodic PM. However, nonperiodic PM is more practical than periodic PM because a system requires more frequent maintenance as time goes by [22].

The general health condition of a repairable system can be measured by its effective age. A PM model that uses effective age is called an age reduction model. Nakagawa [23] introduced improvement factors in the hazard rate and age for a sequential PM policy, and analyzed two imperfect PM models. Related investigations can be found in [7, 17, 18]. Traditional PM models usually assume that the health condition of a repairable system after a PM activity is performed is immediately either as good as new or as bad as old. In this study, a more realistic assumption, that the health condition after a PM activity is performed is somewhere between as good as new and as bad as old, that is an imperfect PM, is made. Furthermore, whenever a breakdown occurs between two successive PM activities, the system undergoes a minimal repair, which only restores the basic operation functions; hence, the failure rate of the repairable system remains unchanged. The

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concept of minimal repair was first introduced in [2] and has been used to develop optimal PM policies for repairable systems for which the breakdown process is known with certainty [16,21,23,24]. Comprehensive surveys of various PM policies for deteriorating single-unit systems have been conducted in the literature [29,30] which reviewed the existing age-dependent PM policies and made some comparisons for them.

In practice, the breakdown process of a system is usually unknown or known with uncertain parameters. In such situations, a Bayesian method that utilities expert opinions and information from a similar system currently in operation can be employed to improve decision making (see [13,26]). Guida et al. [12] conducted a Bayesian inference for the parameters of the Weibull power law intensity function of an NHPP and compared the result with the corresponding maximum likelihood estimation. Mazzuchi and Soyer [19] presented a Bayesian analysis for determining the optimal replacement strategies for both the block replacement protocol with minimal repair and the traditional age replacement protocol. Related investigations can be found in [9,27]. However, studies that have used Bayesian methods often assumed that either one of the parameters of the Weibull power law intensity of an NHPP is known or both parameters are independent [8,9,13,15,19].

Bassin [3] proposed a Bayesian overhaul model in which the breakdown process of the system is modeled by an NHPP with a Weibull power law intensity function, and the two parameters of such an intensity function are assumed to be unknown but conditionally dependent. However, his study focused on the determination of an optimal overhaul interval with consideration of only one perfect PM activity such that the expected total cost per operating hour in an overhaul cycle is minimized. Based on the aforementioned discussion, a more general approach that can flexibly and effectively describe the breakdown process and deal with the problem of scarce data is required for developing a multi-PM strategy for deteriorating repairable systems. In this study, an optimal nonperiodic PM strategy of a deteriorating system, which consists of the determinations of the number of PM activities, the schedule of the PM activities, and the system lifetime (the optimal time to replacement) can be derived by the proposed decision model in which the breakdown process is described by an NHPP with a two-parameter Weibull power law intensity function and a minimal repair is assumed to be performed immediately after each breakdown. Moreover, in addition to the case that the two parameters of the intensity are known, for the case that the two parameters are unknown and not independent, a Bayesian decision framework is considered to tackle the uncertainty of the system failure intensity and facilitate the integrated uses of expert opinions and additional information to determine the prior and posterior optimal PM strategies.

The rest of this article is organized as follows: Section 2 states the fundamental assumptions of the PM model and derives the optimal PM schedule for deteriorating repairable systems for when both of the parameters of the intensity function are given. Section 3 proposes a Bayesian decision approach to derive the optimal PM schedule for deteriorating repairable systems with uncertain failure intensity. Section 4 presents a numerical example with actual data to demonstrate the usefulness of the proposed approach. Sensitivity analyses are carried out to investigate the factors which affect optimal decisions. Finally, concluding remarks are given in Section 5.

2. NONPERIODIC PREVENTIVE MAINTENANCE

The age of a repairable system can be reduced to be younger-than-now (called the effective age) by a PM activity. In this study, a scheduled nonperiodic PM strategy is considered because it is practical for deteriorating repairable systems, which deteriorate with age and require more frequent maintenance over time. An NHPP with the Weibull power law intensity function $\lambda(t) = \alpha\beta t^{\beta-1}$ is assumed to be suitable for describing the deterioration. The deterioration rate (or the shape parameter) β affects how the system deteriorates or improves over time. If $\beta > 1$, the system is deteriorating and breakdowns tend to occur more frequently over time. If $\beta < 1$, the intensity function $\lambda(t)$ is decreasing and the system is improving (i.e., the time between breakdowns tends to increase over time). When $\beta = 1$, the NHPP reduces to a homogeneous Poisson process with intensity α. Note that α (the scale parameter) and β of the intensity function may both be unknown; they are discussed as uncertain states in the Bayesian analysis in Section 3. The planned optimal PM schedule of periodic and nonperiodic PM policies is similar if the deterioration of the repairable system is weak. Hence, a strong deteriorating case is considered in this study (say, $\beta > 2$). The underlying assumptions in this article are as follows:

1. The system has a deterioration process that behaves as an NHPP with a Weibull power law intensity function $\lambda(t) = \alpha\beta t^{\beta-1}$, where $\alpha > 0$ and $\beta > 2$ are the scale parameter and the deterioration rate, respectively, and t is the elapsed time.
2. A PM activity does not restore the system to a state of good-as-new; instead, it restores the system to a state of better-than-now.
3. Any breakdown which occurs between two successive PM activities leads to a minimal repair. The times required for repairs and PM activities are both negligible.
4. The system undergoes $(N-1)$ PM activities during its lifetime T_L and is replaced at the Nth PM activity.

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2.1. Problem Formulation

The notation used in this study are as follows:

- N: number of scheduled nonperiodic PM activities
- x_i: time span between the $(i-1)$th and the ith PM activities, where $i = 1, 2, \ldots, N$
- T_L: $\sum_{i=1}^{N} x_i$; system lifetime
- N_m: expected number of minimal repairs
- y_i: effective age of the system immediately before the ith PM activity; $\boldsymbol{y} = (y_1, y_2, \ldots, y_N)$
- ξ_i: age reduction factor in effective age due to the ith PM activity, where $\xi_i \in (0, 1)$
- c_m: average cost of performing a minimal repair
- c_r: cost of replacing the whole system
- c_p: average cost of performing a PM activity
- $C(\cdot)$: expected cost function (i.e., expected total cost per unit time)
- α: scale parameter of the intensity function
- β: deteriorating rate of the intensity function
- $\boldsymbol{\theta}$: (α, β); parameter vector of the intensity function
- $\pi(\boldsymbol{\theta})$: prior distribution of $\boldsymbol{\theta}$
- $\pi_{\boldsymbol{\theta}|t}(\boldsymbol{\theta})$: posterior distribution of $\boldsymbol{\theta}$

Martorell et al. [20] introduced a practical aging model, the proportional age setback (PAS) model. They showed that the PAS model can fit the imperfect maintenance of repairable systems well. The PAS model assumes that each maintenance activity proportionally reduces the system age to the age immediately before the maintenance being performed (see [10, 11, 17, 23]). We assume that the PAS model can be applied to describe the status of the deteriorating system after PM activities are performed according to the knowledge and experience of the field engineers. for a fixed number of scheduled PM activities, N, the effective ages of the system can thus be derived as:

$$\begin{aligned} y_i &= x_i + \xi_{i-1} y_{i-1} \\ &= x_i + \xi_{i-1} x_{i-1} + \cdots + \xi_{i-1}\xi_{i-2}\cdots\xi_1 x_1, \end{aligned} \tag{1}$$

where $i = 1, 2, \ldots, N$, $0 = \xi_0 < \xi_1 < \cdots < \xi_{N-1} < 1$, $y_0 = 0$, and y_i and $\xi_i y_i$ are the effective ages of the system just before and after the ith PM activity, respectively. For a deteriorating system, a higher effective age indicates a worse health condition of the system. Thus, an increasing sequence of age reduction factor $\{\xi_i\}$ is assumed to reflect the fact that if the costs of performing each PM activity remain unchange, latter PM activities improve the health condition of the system less than initial ones. Since the breakdown process of the system is modeled by an NHPP, if the parameters $\boldsymbol{\theta} = (\alpha, \beta)$ of the intensity function are known by the decision maker, the expected number of breakdowns is given by:

$$\begin{aligned} N_m &= \int_0^{y_1} \lambda(t)\,dt + \int_{\xi_1 y_1}^{y_2} \lambda(t)\,dt + \cdots + \int_{\xi_{N-1} y_{N-1}}^{y_N} \lambda(t)\,dt \\ &= \sum_{i=1}^{N} \int_{\xi_{i-1} y_{i-1}}^{y_i} \lambda(t)\,dt \\ &= \alpha \sum_{i=1}^{N} \left[y_i^{\beta} - (\xi_{i-1} y_{i-1})^{\beta} \right]. \end{aligned} \tag{2}$$

Hence, the expected total cost per unit time for performing nonperiodic PM activities N times is given by:

$$C(N, \boldsymbol{y}|\boldsymbol{\theta}) = \frac{1}{T_L}[c_r + c_m N_m + (N-1)c_p], \tag{3}$$

where $\boldsymbol{y} = (y_1, y_2, \ldots, y_N)$ and the system lifetime is:

$$T_L = \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} y_i - \sum_{i=1}^{N} \xi_{i-1} y_{i-1} = y_N + \sum_{i=1}^{N-1} (1 - \xi_i) y_i.$$

In Eq. (3), the decision variables are $\boldsymbol{y}$ and N. For each given N, $N = 1, 2, \ldots, \boldsymbol{y}$ can be evaluated by minimizing Eq. (3), but the solutions for $\boldsymbol{y}$, are not optimal unless $N = N^*$ where $N^* = \{N | \min_N C(N, \boldsymbol{y}|\boldsymbol{\theta}), N = 1, 2, \ldots\}$. That is, N is given before evaluating y_i, $i = 1, 2, \ldots, N$, and the optimal PM schedule can be obtained by substituting these solutions and their corresponding Ns into Eq. (3) to search for the minimal value of Eq. (3). The found pair of N^* and $\boldsymbol{y}^*$ is thus optimal. More detail discussion is shown in Section 2.2. Moreover, since the repairable system is replaced at the Nth PM activity, the optimal replacement of the entire system would occurs at the time when the effective age of the system equals y_{N^*}, and the system lifetime, which is thus can be computed easily by summing the N^* optimal PM intervals. The expected cost function involves the cost of performing a minimal repair c_m, the cost of undertaking a PM activity c_p, and the cost of replacing the whole system c_r. Note that these three costs can be obtained from a field investigation performed by the engineering department.

2.2. Optimal Solution

The objective of the study is to determine the optimal PM schedule such that the expected total cost per unit time is minimized. A necessary condition for $\boldsymbol{y}$ to minimize the expected cost function is that $\partial C(N, \boldsymbol{y}|\boldsymbol{\theta})/\partial y_i = 0$ for $i = 1, 2, \ldots, N$; therefore, setting the first-order partial derivatives of Eq. (3) with respect to each y_i to zero leads to:

$$(1 - \xi_i)^{-1}[\lambda(y_i) - \xi_i\, \lambda(\xi_i y_i)] = \lambda(y_N) \tag{4}$$

where $i = 1, 2, \ldots, N-1$ and:

$$c_m\, \lambda(y_N) = C(N, \boldsymbol{y}|\boldsymbol{\theta}) \tag{5}$$

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PROPOSITION 1: For a fixed $y_N > 0$, there exits one and only one solution to Eq. (4) ($y_i, i = 1, 2, \ldots, N-1$) if:

a. the intensity function $\lambda(t)$ of an NHPP is continuous and strictly increasing
b. $d\lambda(t)/dt$ is also strictly increasing

PROOF: Suppose that $\lambda(t)$ is continuous and strictly increasing. From Eq. (4), we have the following inequalities:

$$(1-\xi_i)^{-1}[\lambda(0) - \xi_i\lambda(0)] < \lambda(y_N)$$

and

$$(1-\xi_i)^{-1}[\lambda(y_N) - \xi_i\lambda(\xi_i y_N)] > \lambda(y_N).$$

For a fixed y_N, there exists some y_i, $0 < y_i < y_N$, which satisfies Eq. (4). If $\lambda(t)$ is differentiable and $d\lambda(t)/dt$ is strictly increasing, then the derivative of the left hand side of Eq. (4) with respect to y_i is given by:

$$(1-\xi_i)^{-1}\left[\lambda'(y_i) - \xi_i^2\lambda'(\xi_i y_i)\right],$$

where

$$\lambda'(y_i) - \xi_i^2\lambda'(\xi_i y_i) > \left(1-\xi_i^2\right)\lambda'(y_i) > 0.$$

This shows that the left hand side of Eq. (4) is a strictly increasing function of y_i; thus, the solution to Eq. (4) is unique. □

Substituting each solution of Eq. (4) into Eq. (5), we have:

$$\lambda(y_N)\left[y_N + \sum_{i=1}^{N-1}(1-\xi_i)y_i\right] - \sum_{i=1}^{N}\int_{\xi_{i-1}y_{i-1}}^{y_i}\lambda(t)\,dt = \frac{1}{c_m}[c_r + (N-1)c_p]. \quad (6)$$

It should be noted that each $y_i, i = 1, 2, \ldots, N-1$, in Eq. (6) is given by some function of y_N. If there exists a solution y_N to Eq. (6), the expected total cost per unit time is a function of N and can be minimized. That is, for each fixed N, a corresponding nonperiodic PM schedule, which minimizes its corresponding expected cost as in Eq. (3), can be obtained. Hence, the optimal number of PM activities, N^*, can be determined by comparing the obtained expected costs for different Ns and searching for the overall minimal expected cost. The evaluation of $\boldsymbol{y}$ is straightforward. Solving Eq. (4) with respect to y_i, we have:

$$y_i = \left(\frac{1-\xi_i}{1-\xi_i^\beta}\right)^{1/(\beta-1)} y_N, \quad (7)$$

where $i = 1, 2, \ldots, N-1$. Substituting these solutions [i.e., Eq. (7)] into Eq. (6) and solving with respect to y_N, we get:

$$y_N = \left[\frac{c_r + (N-1)c_p}{\alpha(\beta-1)c_m\sum_{j=0}^{N-1}A_j}\right]^{1/\beta}, \quad (8)$$

where $A_j = [(1-\xi_j)^\beta/(1-\xi_j^\beta)]^{1/(\beta-1)}$. Note that Proposition 1 shows that, for a fixed N, these solutions y_i, where $i = 1, 2, \ldots, N$, are unique if $\beta > 2$.

PROPOSITION 2: Let $C(N)$ be an expected cost function that has the form of Eq. (3). Then, $C(N)$ is a convex function with respect to N, and there exists a finite N^* that satisfies the inequalities $C(N^*+1) \geq C(N^*)$ and $C(N^*) < C(N^*-1)$, where N^* is the optimal number of PM activities such that $C(N^*)$ is minimum.

PROOF: From Eq. (5), minimizing the expected cost function is equivalent to minimizing:

$$\lambda(y_N) = \alpha\beta\left[\frac{c_r + (N-1)c_p}{\alpha(\beta-1)c_m\sum_{i=0}^{N-1}A_i}\right]^{(\beta-1)/\beta},$$

or to minimizing:

$$G(N) = \frac{c_r + (N-1)c_p}{\sum_{i=0}^{N-1}A_i},$$

where $N = 1, 2, \ldots$. A necessary condition for the existence of a finite N^* is that N^* satisfies inequalities $G(N^*+1) \geq G(N^*)$ and $G(N^*) < G(N^*-1)$, which implies that:

$$H(N^*) \geq \frac{c_r}{c_p} \quad \text{and} \quad H(N^*-1) < \frac{c_r}{c_p},$$

where

$$H(N) = \begin{cases} A_N^{-1}\sum_{i=0}^{N-1}A_i - (N-1) & \text{if } N = 1, 2, \ldots. \\ 0 & \text{if } N = 0 \end{cases}$$

If A_i is decreasing in i, then $H(N)$ is strictly increasing in N and hence there exists one and only one minimum N^* such that $H(N^*) \geq c_r/c_p$, if $\lim_{N\to\infty}H(N) > c_r/c_p$. To show that A_i is decreasing in i, let $u(x) = (1-x)^\beta/(1-x^\beta)$ then we have:

$$\frac{du(x)}{dx} = \frac{\beta(1-x)^{\beta-1}(x^{\beta-1}-1)}{(1-x^\beta)^2} < 0,$$

for $\beta > 1$ and $0 < x < 1$. That is, $u(x)$ is strictly decreasing from 1 to 0 on $0 < x < 1$ for $\beta > 1$ (see Figure 1). Since $\{\xi_i\}$ is assumed to be an increasing sequence, we have

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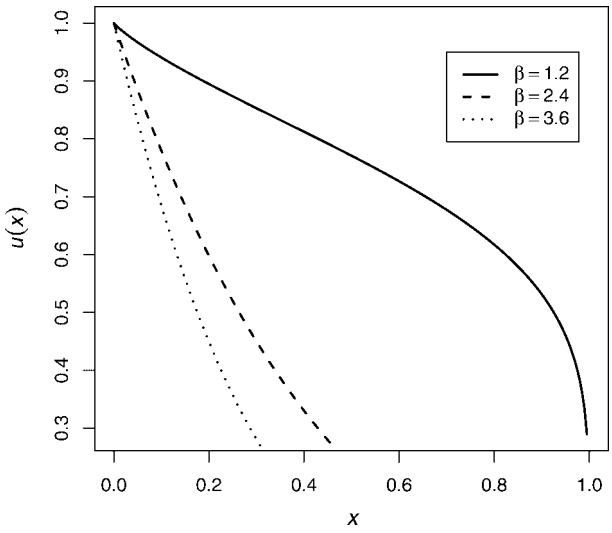

Figure 1. Relationship between $u(x)$ and x for $\beta = 1.2$, 2.4, and 3.6.

$u(\xi_i) > u(\xi_{i+1})$, which implies that $A_i > A_{i+1}$. If $\xi_i \to 1$ as $i \to \infty$, then:

$$\lim_{i\to\infty} A_i = \lim_{x\to 1}[u(x)]^{1/(\beta-1)} = 0.$$

That is, $H(N) \to \infty$ and a finite N^* exists uniquely. Figure 2 graphically shows that $H(N)$ is strictly increasing in N. □

Once N^* is determined, the optimal PM intervals can be computed from $x_i = y_i - \xi_{i-1}y_{i-1}$, where $i = 1, 2, \ldots, N^*$, and y_i are given in Eqs. (7) and (8).

PROPOSITION 3: The length of the PM intervals, x_i, $i = 1, 2, \ldots, N-1$, is marginal decreasing if the age reduction factor, ξ_i, is marginal increasing.

PROOF: By Eqs. (1) and (7), the first $(N-1)$ PM intervals are given by:

$$x_i = \left[\left(\frac{1-\xi_i}{1-\xi_i^\beta}\right)^{1/(\beta-1)} - \xi_{i-1}\left(\frac{1-\xi_{i-1}}{1-\xi_{i-1}^\beta}\right)^{1/(\beta-1)}\right] y_N, \tag{9}$$

where $i = 1, 2, \ldots, N-1$ and y_N is given in Eq. (8) and is a constant for any fixed N. Thus, to show that $\{x_i\}$ is a marginal decreasing sequence is equivalent to show that $\{x_i'\} = \{x_i/y_N\}$ is a marginal decreasing sequence. Suppose that, for $\beta > 1$, $\{[(1-\xi_i)/(1-\xi_i^\beta)]^{1/(\beta-1)}\}$ is a marginal decreasing sequence and $\{\xi_i[(1-\xi_i)/(1-\xi_i^\beta)]^{1/(\beta-1)}\}$ is a marginal increasing sequence then we have the following inequalities:

$$\left(\frac{1-\xi_i}{1-\xi_i^\beta}\right)^{1/(\beta-1)} - \xi_i\left(\frac{1-\xi_i}{1-\xi_i^\beta}\right)^{1/(\beta-1)} < \left(\frac{1-\xi_i}{1-\xi_i^\beta}\right)^{1/(\beta-1)} - \xi_{i-1}\left(\frac{1-\xi_{i-1}}{1-\xi_{i-1}^\beta}\right)^{1/(\beta-1)}$$

and

$$\left(\frac{1-\xi_{i-1}}{1-\xi_{i-1}^\beta}\right)^{1/(\beta-1)} - \xi_{i-1}\left(\frac{1-\xi_{i-1}}{1-\xi_{i-1}^\beta}\right)^{1/(\beta-1)} > \left(\frac{1-\xi_i}{1-\xi_i^\beta}\right)^{1/(\beta-1)} - \xi_{i-1}\left(\frac{1-\xi_{i-1}}{1-\xi_{i-1}^\beta}\right)^{1/(\beta-1)},$$

which implies that:

$$(1-\xi_i)\left(\frac{1-\xi_i}{1-\xi_i^\beta}\right)^{1/(\beta-1)} < x_i' < (1-\xi_{i-1})\left(\frac{1-\xi_{i-1}}{1-\xi_{i-1}^\beta}\right)^{1/(\beta-1)}.$$

Thus, if $0 < \xi_i < 1, i = 1, 2, \ldots, N-1$, is marginal increasing, then $\{x_i\}$ is a marginal decreasing sequence. Let $v(z) = (1-z)/(1-z^\beta)$ and $w(z) = (z^{\beta-1} - z^\beta)/(1-z^\beta)$, respectively. Next, we show that $\{[(1-\xi_i)/(1-\xi_i^\beta)]^{1/(\beta-1)}\}$ is a marginal decreasing sequence and $\{\xi_i[(1-\xi_i)/(1-\xi_i^\beta)]^{1/(\beta-1)}\}$ is a marginal increasing sequence by showing that $v(z)$ is

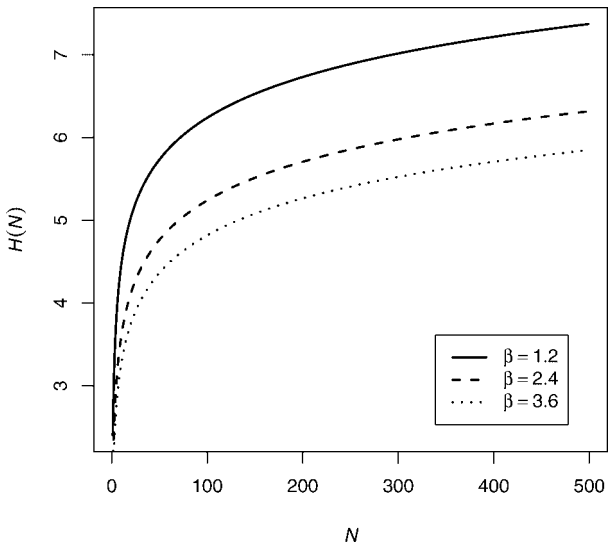

Figure 2. Relationship between $H(N)$ and N for $\beta = 1.2$, 2.4, and 3.6.

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a strictly decreasing function and $w(z)$ is a strictly increasing function. For $\beta > 1$ and $0 < z < 1$, we have the first derivative of $v(z)$ with respect to z, which is given by:

$$\frac{\mathrm{d}v(z)}{\mathrm{d}z} = \frac{(1-\beta)z^{\beta} + (\beta z^{\beta-1} - 1)}{(1-z^{\beta})^2} < 0. \qquad (10)$$

since $\beta z^{\beta-1} < 1$. In a similar manner, since $\beta(1-z) > 1-z^{\beta}$ for $\beta > 1$ and $0 < z < 1$, we have:

$$\frac{\mathrm{d}w(z)}{\mathrm{d}z} = \frac{z^{\beta-2}[\beta(1-z) - (1-z^{\beta})]}{(1-z^{\beta})^2} > 0, \qquad (11)$$

and $w(z)$ is thus a strictly increasing function. Eqs. (10) and (11) imply that, for $\beta > 1$ and $0 < \xi_i < 1$, $i = 1, 2, \ldots, N-1$, is marginal increasing, $\{[(1-\xi_i)/(1-\xi_i^{\beta})]^{1/(\beta-1)}\} = \{[v(\xi_i)]^{1/(\beta-1)}\}$ is a marginal decreasing sequence and $\{\xi_i[(1-\xi_i)/(1-\xi_i^{\beta})]^{1/(\beta-1)}\} = \{[w(\xi_i)]^{1/(\beta-1)}\}$ is a marginal increasing sequence, respectively. □

Since the costs of performing imperfect PM activities are assumed to be the same, the degrees of restoration would decrease for successive PM activities due to system deterioration. Proposition 3 indicates that, for a deteriorating repairable system, the successive PM intervals, except for the last one, would tend to be shorter and shorter when imperfect PM activities are performed.

3. BAYESIAN ANALYSIS FOR OPTIMAL PM SCHEDULE

The primary concern in determining an optimal PM strategy for a deteriorating repairable system is the uncertainty about its deterioration status. Since an NHPP with a Weibull power law intensity is used to model the deterioration process in this study, the parameters of the intensity function, which may usually be unknown have to be accurately estimated to gain more insights for the follow-up decision making. Having collected a sufficiently large amount of observed data, Bayesian analysis and traditional statistical analysis may lead to similar results. However, in the case of insufficient available data, Bayesian analysis becomes more attractive for decision makers because of the fact that it allows for incorporating their operating experiences or technical knowledge in the estimation process. In fact, it is quite common that the data for reliability analysis are scarce or seriously censored. In general, the prior knowledge of a decision maker often involves some physical understandings about the system, and the real problem turns out to be the quantification process, which converts such the prior information into a degree of belief about the true values of the parameters. Meanwhile, if empirical data are available and collected, a more reliable estimation of the parameters can be achieved by considering the additional empirical data. Note that empirical data are usually collected from a similar system and the posterior (updated prior) distribution is thus able to incorporate certain degree of operating information about actual deterioration.

With prior knowledge and the empirical data, an effective analysis can be achieved using the Bayesian updating process. The Bayesian decision process can be divided into two phases: the prior analysis and the posterior analysis. In the first phase, decision makers examine the assumptions made about the breakdown process, and work with domain experts to specify the joint prior distribution of α and β, which describes the uncertainty of deterioration. The relevant costs of a PM activity are then investigated to derive the optimal PM schedule. In the second phase, the collected sample data are taken into consideration to improve the quality of decision making. The results of the posterior analysis integrate both expert opinions and sampling information. The basic elements for the Bayesian analysis process in this study are as follows:

- State space $\Theta = \{\boldsymbol{\theta} = (\alpha, \beta) | \alpha > 0, \beta > 0\}$.
- Action space $\mathcal{A} = \{(N, \boldsymbol{y}) | \boldsymbol{y} = (y_1, y_2, \ldots, y_N), N = 1, 2, \ldots\}$, where N denotes the number of PM activities and $\boldsymbol{y}$ is the scheduled PM activities.
- Loss function $C(\cdot)$; a real function defined on $\Theta \times \mathcal{A}$. The costs incurred if N nonperiodic PM activities are chosen under the state of nature $\boldsymbol{\theta}$ given by $C(N, \boldsymbol{y}, \boldsymbol{\theta})$.
- Sample space $\mathcal{S}$; additional information available to be collected (e.g., successive breakdown times from similar systems).

3.1. Prior and Posterior Analyses

Suppose that expert opinions and past experience can be used to obtain an appropriate prior distribution $\pi(\boldsymbol{\theta})$ to perform the Bayesian analysis. The Bayes estimator of the expected total cost per unit time relative to $\pi(\boldsymbol{\theta})$ is given by:

$$\mathrm{E}_{\boldsymbol{\theta}}[C(N, \boldsymbol{y}, \boldsymbol{\theta})] = \frac{1}{T_{\mathrm{L}}}[c_{\mathrm{r}} + c_{\mathrm{m}}\,\mathrm{E}_{\boldsymbol{\theta}}(N_{\mathrm{m}}) + (N-1)c_{\mathrm{p}}], \quad (12)$$

where

$$\begin{aligned}\mathrm{E}_{\boldsymbol{\theta}}(N_{\mathrm{m}}) &= \sum_{i=1}^{N} \mathrm{E}_{\boldsymbol{\theta}}\left[\int_{\xi_{i-1}y_{i-1}}^{y_i} \lambda(t)\,dt\right] \\ &= \sum_{i=1}^{N} \mathrm{E}_{\boldsymbol{\theta}}\left(\alpha y_i^{\beta}\right) - \sum_{i=1}^{N-1} \mathrm{E}_{\boldsymbol{\theta}}\left[\alpha(\xi_i y_i)^{\beta}\right]\end{aligned}$$

One might confuse the matter that the expectation on the expected cost function [on the left-hand side of Eq. (12)]. Recall that the cost function involves the expected number

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of breakdowns and the average costs of repair; we therefore call $C(\cdot)$ the "expected total cost per unit time" (or "expected cost function") rather than the "total cost per unit time" in this article. In the Bayesian decision framework, the parameters of the intensity function are treated as random variables, say $\boldsymbol{\theta} \sim \pi(\boldsymbol{\theta})$, and an estimator that has the smallest risk for values of $\boldsymbol{\theta}$ is used to estimate the expected cost function $C(N, \boldsymbol{y}, \boldsymbol{\theta})$. It coincides with the objective of this study and can be done by computing the expected loss of an estimator of $C(N, \boldsymbol{y}, \boldsymbol{\theta})$, averaged over values of $\boldsymbol{\theta}$ with respect to $\pi(\boldsymbol{\theta})$, and choose an estimator with minimum expected loss (i.e., the Bayes estimator).

The algorithm for finding the optimal nonperiodic PM schedule of the prior analysis is summarized as follows:

Step 1. Set $N = 1$ and solve $\mathrm{dE}_{\boldsymbol{\theta}}[C(N = 1, y_1, \boldsymbol{\theta})]/\mathrm{d}y_1 = 0$ with respect to y_1. By substituting the solution into Eq. (12) then we have the value of $\mathrm{E}_{\boldsymbol{\theta}}[C(N = 1)]$.

Step 2. Let $N = N + 1$ and solve the system of $\partial \mathrm{E}_{\boldsymbol{\theta}}[C(N + 1, \boldsymbol{y}, \boldsymbol{\theta})]/\partial \boldsymbol{y} = \mathbf{0}$ with respect to $\boldsymbol{y}$, where $\boldsymbol{y} = (y_1, \ldots, y_{N+1})$. By substituting these solutions into Eq. (12) we obtain the value of $\mathrm{E}_{\boldsymbol{\theta}}[C(N + 1)]$.

Step 3. If $\mathrm{E}_{\boldsymbol{\theta}}[C(N + 1)] > \mathrm{E}_{\boldsymbol{\theta}}[C(N)]$ then the optimal number of PM activities, say $N^* = \{N | \min_N \mathrm{E}_{\boldsymbol{\theta}}[C(N, \boldsymbol{y}, \boldsymbol{\theta})]$, where $N = 1, 2, \ldots\}$, is attained and stop; otherwise, return to Step 2.

Step 4. Compute x_i from $x_i = y_i - \xi_{i-1} y_{i-1}$, where $i = 1, 2, \ldots, N^*$.

Similarly, if an adequate sample $\boldsymbol{t} = (t_1, t_2, \ldots, t_n)$ is drawn from a similar system to form a posterior distribution $\pi_{\boldsymbol{\theta}|\boldsymbol{t}}(\boldsymbol{\theta})$, then the Bayes estimator of $C(N, \boldsymbol{y}, \boldsymbol{\theta})$, which can be obtained by minimizing the expected loss relative to the posterior distribution $\pi_{\boldsymbol{\theta}|\boldsymbol{t}}(\boldsymbol{\theta})$, is given by:

$$\mathrm{E}_{\boldsymbol{\theta}|\boldsymbol{t}}[C(N, \boldsymbol{y}, \boldsymbol{\theta})] = \frac{1}{T_{\mathrm{L}}}[c_{\mathrm{r}} + c_{\mathrm{m}} \mathrm{E}_{\boldsymbol{\theta}|\boldsymbol{t}}(N_{\mathrm{m}}) + (N - 1)c_{\mathrm{p}}] \tag{13}$$

where $\mathrm{E}_{\boldsymbol{\theta}|\boldsymbol{t}}(\cdot)$ denotes the expectation calculated from the posterior distribution $\pi_{\boldsymbol{\theta}|\boldsymbol{t}}(\boldsymbol{\theta})$. The algorithm for finding the optimal nonperiodic PM schedule of the posterior analysis is the same as that for the prior analysis except that the Bayes estimator of $C(N, \boldsymbol{y}, \boldsymbol{\theta})$, say $\mathrm{E}_{\boldsymbol{\theta}}[C(N, \boldsymbol{y}, \boldsymbol{\theta})]$, is replaced with $\mathrm{E}_{\boldsymbol{\theta}|\boldsymbol{t}}[C(N, \boldsymbol{y}, \boldsymbol{\theta})]$. Note that when the expected cost function is complex, the optimal $\boldsymbol{y}$ may not be analytically obtainable. In this case, the algorithm becomes a numerical procedure and standard numerical techniques are required in Steps 1 and 2.

3.2. Natural Conjugate Prior Distribution

Bayesian decision analysis is typically not easy to perform because of the difficulty of deriving the posterior distribution. In this study, the analysis is especially difficult because the proposed approach deals with more than one variable in the state space. Huang and Bier [14] proposed a nature conjugate prior distribution for the Weibull power law failure model for repairable systems in the form:

$$\pi(\boldsymbol{\theta}) = K(\alpha\beta)^{b-1}[d^b \exp(-a)]^{\beta-1} \exp(-\alpha c d^{\beta}), \tag{14}$$

where $K = [\Gamma(b)]^{-2}(acd)^b \exp(-a)$. The major benefit of this natural conjugate prior distribution is that it allows a straightforward analysis. The four parameters in Eq. (14) (i.e., a, b, c, and d) can be chosen to achieve a suitable joint distribution by employing the investigated prior marginal moments [14]. Using Eq. (14) as the prior distribution of $\boldsymbol{\theta}$, it is relatively easy to derive the prior expectation of interest using the bivariate transformation technique, which is given by:

$$\mathrm{E}_{\boldsymbol{\theta}}(\alpha y^{\beta}) = \frac{ba^b}{c}[a + \ln(d) - \ln(y)]^{-b}.$$

Thus, Eq. (12) can be rewritten as:

$$\begin{aligned}\mathrm{E}_{\boldsymbol{\theta}}[C(N, \boldsymbol{y}, \boldsymbol{\theta})] = \frac{1}{T_{\mathrm{L}}} \Bigg\{ & c_{\mathrm{r}} + (N - 1)c_{\mathrm{p}} \\ & + \frac{c_{\mathrm{m}} ba^b}{c} \Bigg\{ \sum_{i=1}^{N} [a + \ln(d) - \ln(y_i)]^{-b} \\ & - \sum_{i=1}^{N-1} [a + \ln(d) - \ln(\xi_i y_i)]^{-b} \Bigg\} \Bigg\}\end{aligned} \tag{15}$$

The prior analysis can be performed by calculating $\mathrm{E}_{\boldsymbol{\theta}}[C(N, \boldsymbol{y}, \boldsymbol{\theta})]$ in Eq. (15), where the four parameters are specified by the decision makers with their prior knowledge about the system, and then employing the algorithm shown in Section 3.1. The optimal prior decision of the PM schedule can thus be determined. In Step 1 of the algorithm, $\partial \mathrm{E}_{\boldsymbol{\theta}}[C(N, \boldsymbol{y}, \boldsymbol{\theta})]/\partial y_i = 0$ for $i = 1, 2, \ldots, N$ can be derived as:

$$\begin{aligned}\frac{1}{(1 - \xi_i) y_i} \Big\{ & [a + \ln(d) - \ln(y_i)]^{-(b+1)} \\ & - [a + \ln(d) - \ln(\xi_i y_i)]^{-(b+1)} \Big\} \\ & = \frac{1}{y_N} [a + \ln(d) - \ln(y_N)]^{-(b+1)},\end{aligned} \tag{16}$$

where $i = 1, 2, \ldots, N - 1$ and

$$\frac{c_{\mathrm{m}} b^2 a^b}{c\, y_N} [a + \ln(d) - \ln(y_N)]^{-(b+1)} = \mathrm{E}_{\boldsymbol{\theta}}[C(N, \boldsymbol{y}, \boldsymbol{\theta})], \tag{17}$$

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Table 1. Optimal PM schedules for non-Bayesian, prior, and posterior analyses.

	Optimal PM Intervals[a]											
	N^*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	T_L
Non-Bayesian	10	0.500	0.314	0.280	0.265	0.257	0.252	0.248	0.245	0.243	0.392	2.995
Prior	7	0.416	0.274	0.246	0.234	0.228	0.224	0.469				2.092
Posterior	5	0.281	0.173	0.154	0.146	0.288						1.042

[a]In 1000s of operating hours.

where $E_{\boldsymbol{\theta}}[C(N,\boldsymbol{y},\boldsymbol{\theta})]$ is given in Eq. (15). Unfortunately, because of the complexity of the expected cost function, the solutions to Eqs. (16) and (17) have no closed-form expressions. Numerical techniques are required to evaluate $\boldsymbol{y}$.

Suppose that the further investigation is undertaken for updating the estimation of $\boldsymbol{\theta}$, and that n breakdown times $\boldsymbol{t} = (t_1, t_2, \ldots, t_n)$ are collected, then the posterior distribution of $\boldsymbol{\theta}$, say $\pi_{\boldsymbol{\theta}|\boldsymbol{t}}(\boldsymbol{\theta})$, can be obtained without further computation using the property of the natural conjugate family, which is given by:

$$\begin{aligned}\pi_{\boldsymbol{\theta}|\boldsymbol{t}}(\boldsymbol{\theta}) &\propto f(\boldsymbol{t}|\boldsymbol{\theta})\,\pi(\boldsymbol{\theta}) \\ &= K'(\alpha\beta)^{b+n-1}\left(d^b \exp(-a)\prod_{i=1}^{n} t_i\right)^{\beta-1} \\ &\quad \times \exp\left(-\alpha\left(cd^{\beta} + t_n^{\beta}\right)\right), \quad (18)\end{aligned}$$

where $f(\boldsymbol{t}|\boldsymbol{\theta}) = (\alpha\beta)^n(\prod_{i=1}^{n} t_i)^{\beta-1}\exp(-\alpha t_n^{\beta})$ is the sampling distribution [3, 6] and K' is a normalizing factor that ensures that the posterior distribution sums to unity. Note that the derivation of the sampling distribution (likelihood function) is based on the n observed breakdown times of a similar deteriorating system, which undergos no PM activity within a replacement (or overhaul) cycle. The sampling breakdown times are facilitate to gaining insights about the true deteriorating to enhance the prior estimation of $\boldsymbol{\theta}$ (i.e., α and β). In case that the empirical data are collected from a similar system, which has performed several imperfect PM activities, the only useful data are the breakdown times that are observed before the first PM activity. Unfortunately, since the posterior distribution in Eq. (18) is much more complicate than the prior distribution in Eq. (14), the expected number of breakdowns in posterior analysis, $E_{\boldsymbol{\theta}|\boldsymbol{t}}(N_m)$, cannot be obtained in a closed-form expression. However, by using numerical integration and employing the algorithm shown in Section 3.1, the posterior analysis can be carried out with both the prior knowledge and the sampling information; the optimal posterior decision of the nonperiodic PM schedule can thus be determined. Note that a PM strategy can be updated dynamically when the failure data for the current system is available. However, in general, decision makers create a PM schedule before operating the system in the planning stage.

4. NUMERICAL EXAMPLE

In this section, air conditioner failure data from [25] is used to illustrate the proposed Bayesian decision model. The failure data is a record of failure times before 1000 hours of the air conditioning system of ten members of a fleet of Boeing 720 jet airplanes. The data can be reasonably modeled by an NHPP with the Weibull power law intensity, as shown in [5]. To obtain the optimal PM schedule, we need to determine the age reduction factor, the relevant costs in the expected cost function, and the four parameters in Eq. (14). Suppose that, using the engineers' past experience and knowledge, the cost ratios are given as $c_m/c_p = 2.8$ and $c_r/c_p = 3.4$, and the age reduction factor $\xi_i = i/(2i+1)$, which marginally increases with the frequency of PM. If decision makers are fairly confident that the parameters of the intensity function are $\alpha = 2$ and $\beta = 2.8$, the non-Bayesian analysis described in Section 2 is appropriate (results are summarized in Table 1). However, if decision makers are not sure of the exact values of $\boldsymbol{\theta}$, the Bayesian decision analysis is a better choice for determining the optimal PM schedule. In such a case, suppose that the natural conjugate prior distribution with $E(\alpha) = 2$, $\mathrm{Var}(\alpha) = 2.5$, $E(\beta) = 2.8$, and $\mathrm{Var}(\beta) = 0.4$ is suggested as the joint distribution of $\boldsymbol{\theta}$ based on the experts' prior knowledge and investigation. With the four prior marginal moments assumed earlier, the four parameters for the natural conjugate prior distribution in Eq. (14) can be obtained from [14]; they are $(a, b, c, d) = (7.000, 19.600, 0.422, 3.383)$.

Using the algorithm mentioned in Section 3.2, we obtain the optimal number of PM activities $N^* = 7$ (see Fig. 3) and the system lifetime $T_L = 2.092$ (in thousands of operating hours). The corresponding optimal PM intervals are summarized in Table 1. In the posterior analysis, we update the information on $\boldsymbol{\theta}$ based on the failure data from plane #8045 in [5]. The successive failure times, in thousands of operating hours, of the air conditioner are $\boldsymbol{t} = (0.102, 0.311, 0.325, 0.382, 0.436, 0.468, 0.535, 0.594, 0.728, 0.880, 0.907, 0.921)$. The first failure occurred at 102 hours of operation and the system survived for an additional 209 hours of operation because of the minimal repair required after failures. By substituting $E_{\boldsymbol{\theta}}[C(N,\boldsymbol{y},\boldsymbol{\theta})]$ with $E_{\boldsymbol{\theta}|\boldsymbol{t}}[C(N,\boldsymbol{y},\boldsymbol{\theta})]$, the algorithm can be similarly performed. The results of the non-Bayesian and the Bayesian analyses are summarized in Table

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1. It should be noted that $\boldsymbol{\theta}$ can be updated using the failure data from an arbitrary plane in [5]. The failure data from plane #8045 were selected for illustration as they have the strongest deterioration in the posterior analysis.

Figure 3 shows a comparison of the analyses. Considering uncertainties in estimating $\boldsymbol{\theta}$, the prior analysis is relatively conservative about the deterioration of the system. As shown in Table 1, the optimal number of PM activities, N^*, in the prior analysis is less than that in the non-Bayesian analysis. Comparing the optimal PM schedule of the non-Bayesian analysis with that of the prior analysis, the latter suggests replacing the entire system early to avoid severe breakdowns in the future. In the Bayesian analysis, the collected data support the adoption of fewer PM activities, whereas the prior analysis supports the adoption of more PM activities. However, each optimal PM interval x_i in the posterior analysis is shorter than that of the prior analysis. The collected data indicate a stronger deterioration than was assumed by the prior distribution. A higher expected number of minimal repairs within the system lifetime in the posterior analysis is shown in Fig. 3. There is a significant upward shift in the expected total cost curves as well as a slight left shift in N^*. From Table 1, x_i decreases for all i values except for the last one in both non-Bayesian and Bayesian analyses. This shows that it is reasonable to perform more frequent PM activities with age; however, since the system should be replaced at the last PM activity, it is better to perform the last activity as late as possible.

Some factors affect the optimal number of nonperiodic PM activities. The prior expectations of α and β were investigated. Table 2 summarizes the prior analysis for various combinations of the expectations and the variances of α and β. When $\mathrm{E}(\alpha)$ and $\mathrm{E}(\beta)$ increase, the probability of system breakdown increases. This indicates that the system is more risky, and consequently, more PM activities are needed, which results in higher expected total cost. Since β is the power of the intensity function, the optimal PM activities are more sensitive to $\mathrm{E}(\beta)$ than $\mathrm{E}(\alpha)$, which means that the prior estimation of $\mathrm{E}(\beta)$ requires more attention from managers.

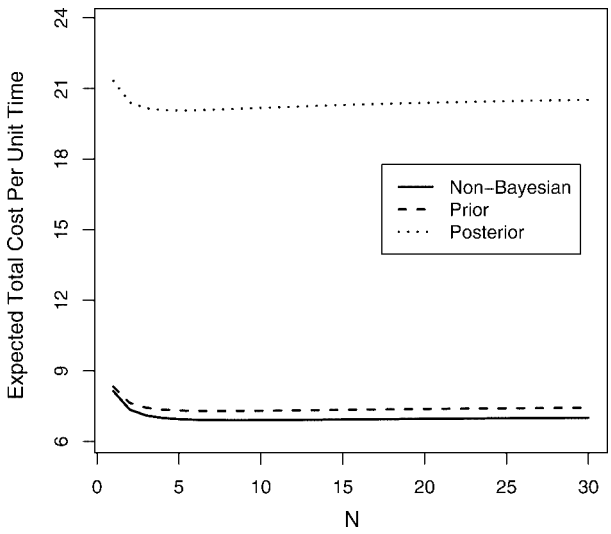

Figure 3. Relationship between the expected total cost function and the number of PM activities.

Table 2. Summary of the prior analysis. (The cost ratios are given as $c_\mathrm{m}/c_\mathrm{p} = 2.8$ and $c_\mathrm{r}/c_\mathrm{p} = 3.4$.)

$\mathrm{E}(\alpha)$	$\mathrm{Var}(\alpha)$	$\mathrm{E}(\beta)$	$\mathrm{Var}(\beta)$	N^*	T_L
1.6	2.5	2.80	0.40	6	1.999
1.8	2.5	2.80	0.40	7	2.146
2.0	2.5	2.80	0.40	7	2.092
2.2	2.5	2.80	0.40	8	2.262
2.4	2.5	2.80	0.40	8	2.228
2.0	1.5	2.80	0.40	8	2.294
2.0	2.0	2.80	0.40	7	2.087
2.0	2.5	2.80	0.40	7	2.092
2.0	3.0	2.80	0.40	7	2.097
2.0	3.5	2.80	0.40	7	2.101
2.0	2.5	2.24	0.40	5	1.843
2.0	2.5	2.52	0.40	6	1.938
2.0	2.5	2.80	0.40	7	2.092
2.0	2.5	3.08	0.40	8	2.275
2.0	2.5	3.36	0.40	9	2.476
2.0	2.5	2.80	0.24	8	2.284
2.0	2.5	2.80	0.32	7	2.080
2.0	2.5	2.80	0.40	7	2.092
2.0	2.5	2.80	0.48	7	2.102
2.0	2.5	2.80	0.56	7	2.113

To see the influence of the prior variances of α and β on N^*, the increment of variances in Table 2 is set to be twice the increment of expectations. As can be seen in Table 2, the variances increase from -40% to 40% based on the values used in the prior analysis (i.e., $\mathrm{Var}(\alpha) = 2.5$ and $\mathrm{Var}(\beta) = 0.4$). Similarly, the expected number of minimal repairs increases with $\mathrm{Var}(\alpha)$ and $\mathrm{Var}(\beta)$; however, because high variances cause the system to deteriorate rapidly, the expected number of breakdowns cannot be reduced efficiently by performing more frequent PM activities. Instead, replacing the whole system before serious deterioration is suggested.

The various related costs have an impact on N^* and therefore influence the expected total cost. Figure 4 shows the relationship between N^* and the three related costs, and Fig. 5 shows the relationship between the expected total cost per unit time and the three related costs. In the case when the average cost of a minimal repair is significantly greater than the average cost of a PM activity, which indicates that performing PM is effective and can result in relatively lower costs, the optimal number of PM activities increases. However, in the case when these two costs are similar, it would be meaningless to perform PM, so the optimal number of PM activities decreases. As shown in Fig. 4, the number of PM activities increases

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when c_m increases. The increase of PM activities reduces the frequency of minimal repairs, but it does not slow down the increase of the expected total cost per unit time while deterioration of the system is significantly serious. Figure 5 shows that the expected total cost function is more sensitive to the average cost of minimal repair. Therefore, the estimation of c_m should receive more attention from managers.

5. CONCLUSION

This article deliberated the ways to develop an appropriately PM policy for deteriorating repairable systems. Upon operating a deteriorating system, two models for determining, in advance, the optimal nonperiodic PM schedule, which minimizes the expected total cost per unit time, were studied. The proposed approaches provide manages with decision making techniques in planning an proper PM strategy in the light of all relevant information about the system deterioration.

One thing particularly noteworthy is that the deterioration of a repairable system is dynamic and may be affected by its daily operating condition, and the optimal PM schedule obtained in advance by the proposed models should thus be adjusted to some extent in accordance with the changing working environment. The results of the sensitivity analyses for investigating the impacts of the parameters of the intensity function and the costs of PM activities on the optimal PM strategy can provide managers with insights about the critical factors to which more attention should be paid in the planning stage. However, it may be pragmatic that the experts are not able to correctly state the age reduction factors and the parameters of the deteriorating intensity. In such a case, getting more experienced experts and spending extra time for more in-depth investigation would certainly improve the accuracy of estimation and thus enhance the decision quality.

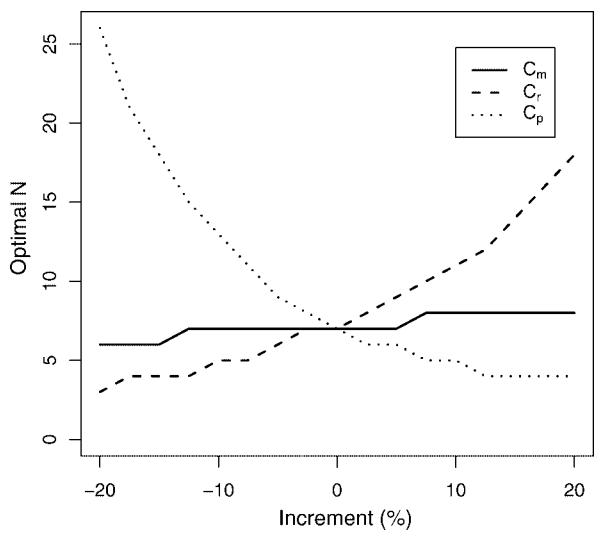

Figure 4. Effects of the related costs on the optimal number of PM activities.

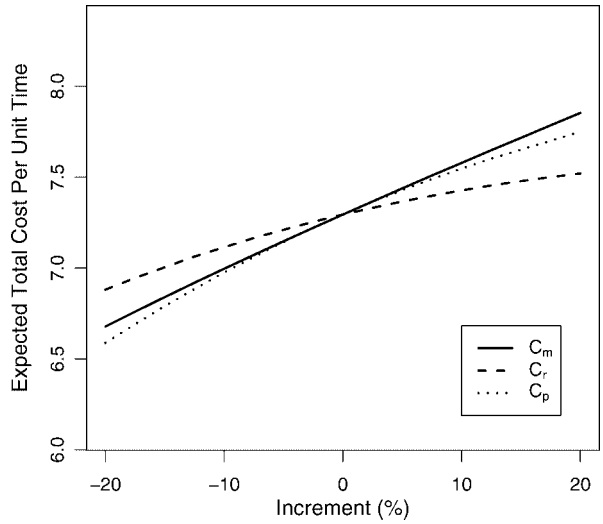

Figure 5. Effects of the related costs on the expected total cost per unit time.

This study can be extended to other breakdown processes or alternative prior distributions can be used. However, the analysis may become complicated. Future work could refine the proposed model by considering the cost of minimal repair as a random variable, since the repair cost is subject to various degrees of malfunction. A nonhomogeneous compound Poisson process might be suitable for this analysis.

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